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Enlightenment and Application of Infinite Rapport Structure in Architectural Design

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Abstract. The infinite rapport structure is continuous repetition of a unit to fill its own space based on certain rhythm arranged. It has long been widely used in decoration of Architecture. This article makes a brief introduction of Geometrical Theory of Crystal Structure, studies the types and features of infinite rapport structure with different Dimension periodicity, and presents some typical design cases in architecture.

Introduction

It's common to use tilling graphics[1] as space decoration of architecture. It's widely used in ancient Chinese window grille, Islamic religious wall decoration, the cornice decoration of Ancient Greek hieron and pavement in Egypt, which has increased the brilliant color for the architectural art. The decoration graphics, both complex and ordered, are just like says in Hegel's history of philosophy: "beauty is included in the volume and the order." Particularly the amazing tilling geometrical graphics in Islamic architecture always astonish the world.

When appreciating and praising the graphics, architects will naturally try to find the mystery and ask themselves : whether the craftsmen created the tilling graphics by accident? And whether there are some secret mathematic methods behind those mysterious and gorgeous graphics? Why can those graphics vary in various forms, but at the same time they also follow the extremely rigorous order?

In fact, there are many wonderful objects with infinite rapport structure in the nature, for example the shape of crystal, snowflake, hexagonal honeycomb, shape of bubble packing and so on. Obviously, the infinite rapport structure is more popular in nature, which has the special features of superiority compared with the other structures.

Indeed, infinite rapport structure follows certain geometry principle. Crystal is a natural kind of solid matter with infinite rapport structure. Through understanding the geometry theory of crystal structure, we may grasp general rule of infinite rapport object. People abstracted a set of complete geometry principle from experiment of atom arrangement of crystal structure. It's called Geometrical Theory of Crystal Structure, which is the common geometrical theory followed by all the infinite rapport structures. We can grasp the general rule of infinite rapport, through learning the geometrical theory of crystal structure. There are 7 types of 1D infinite rapport structures(band tilling graphic), 17 types of 2D infinite rapport structures (plane tilling graphic) and 230 types of 3D infinite rapport structures(spatial tilling graphic) [1]. They are shown in Table 1.

[1] Regular surface cutting(or spatial cutting) is called tilling graphics, and tilling graphic is a permutation of gapless closed graphic without any overlapping.

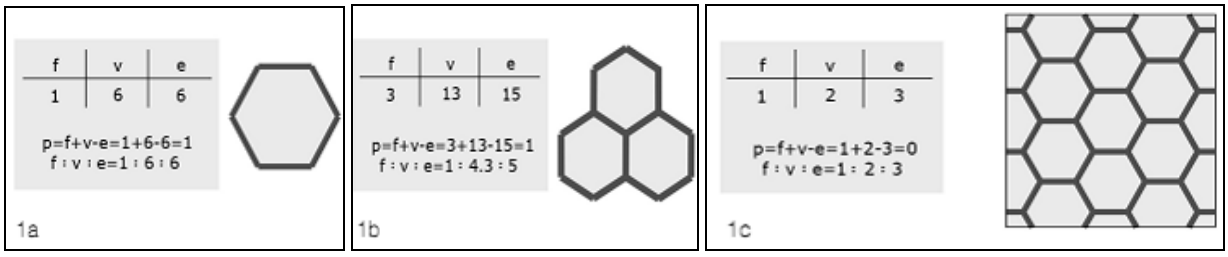
Table 1 Types of infinite rapport structure

Dimension / Symmetry Operations	1D rapport Strruction $R_1^{(1)}$	1D Rapport Strruction $R_1^{(2)}$		2D Rapport Strruction $R_2^{(2)}$		3D Rapport Strruction $R_3^{(3)}$	
Point Group	2	5		10		32	
NonPoint	None	Translation		Translation		Translation Screw-rotation Glide-reflection	
Translational Periodicity	1	1		5		14	
Type	2	7	Point Types 5	17	Point Types 13	320	Point Types 73
			NonPoint Types 2		NonPoint Types 4		NonPoint Types 157

Stability, Efficiency and Economy—Infinite Rapport Structure Reflect Maximum Economic Principle in Nature

In the nature, evolution always is in the simplest and the most efficient way; that is, the maximum economic principle –certainly pay the minimum cost to make the transformation [2]. Infinite rapport structure follows the principle of nature, namely, put the same unite in closely repeating permutation and connection so that they support and limit each other to reach a stable balance. It has defined ability to resist destruction from outside and its balance changes only if the force of destruction reached its defined value (such as every crystal has the defined melting point). Meanwhile, it has the feature that entirety is bigger than parts, namely, connected entirety structure has higher efficiency and consumes less resource than its dispersive state.[1] It is shown in Fig. 1 as follow.

Solidity is the most important thing for a building meanwhile economical efficiency and high efficiency are also necessary. So it is the most reasonable solution to design the large-span building that use of steel to create an infinite rapport structure. Because its weight is very light and its bear of load is heavy.



Suppose **f** is the number of plane, **v** is the number of vertex, **e** is the number of line, **p** is the number of Euler characteristic. When the number of unit is increasing, the efficiency of whole entirety will constantly enhance, and p suddenly change into 0.

Fig.1 Efficiency of Rapport Structure

Analysis of Application Case of Infinite Rapport Structure in Contemporary Architecture

The infinite rapport structure is used in contemporary architecture design which is never stopped. With deeply understanding of properties of infinite rapport structure, architects have been engaged in a variety of exploration and try to apply it better.

[1]Infinite rapport structure has some characteristics of emergence, such as entirety is larger than the summation of every part. The essence of emergence is from small to big, and from simple to complicate. The phenomenon of emergence is always happened explosively. For example, when 7 hexagons connected, its Euler Characteristic is changed. Concerning more information about emergence, readers can refer *Emergence: From Chaos to Order*, John Holland(translated by Chen Yu, Shanghai Century Press Group,2006)

The infinite rapport structure is extremely suitable for application to design building skins. In the present, the building skin as the focus of architectural design is more paid attention by architect.

Paris Arab World Cultural Center

Paris Arab World Cultural Center is a successful case of application of the infinite rapport structure in building skin (Fig. 2).

It was designed by Jean Nouvel, famous French architect. The architect did not select the simple way that using beancurd sheets to control light, but rather select automatic light control device named photosensitive light-fold control equipment as the building skin to control light. The design of building skin was inspired by tilling graphics. It combines the traditional culture and modern technology together. The building skin has dazzling effectiveness and practical function. It can be said that the skin design of Arab World cultural Center is a modern expression of the Arab world decorative art.

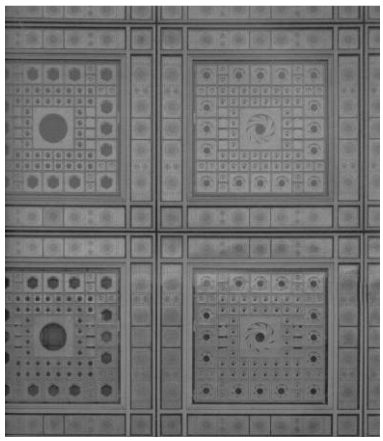


Fig.2 Skin of Paris Arab World Cultural Center

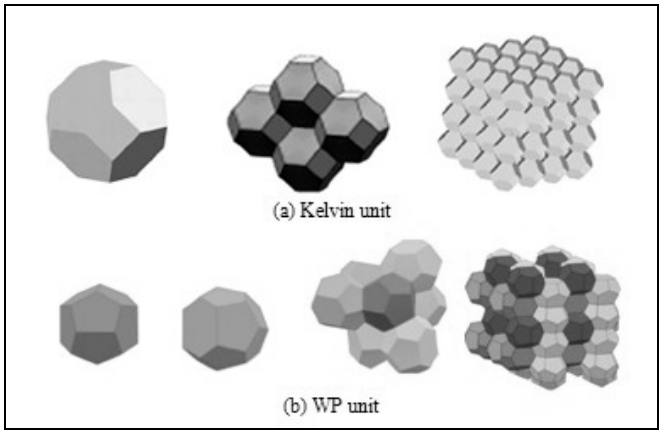


Fig.3 Comparison between Kelvin Unit and WP Unit

Beijing Olympic National Aquatics Center (Water Cube)

The idea of National Aquatics Center (Water Cube) is a bubble packing cube. But it is not practical to simulate the shape of random bubbles in nature. Therefore, it's necessary to find a form which has highly efficient structure and can be easily constructed to simulate the shape of bubbles.

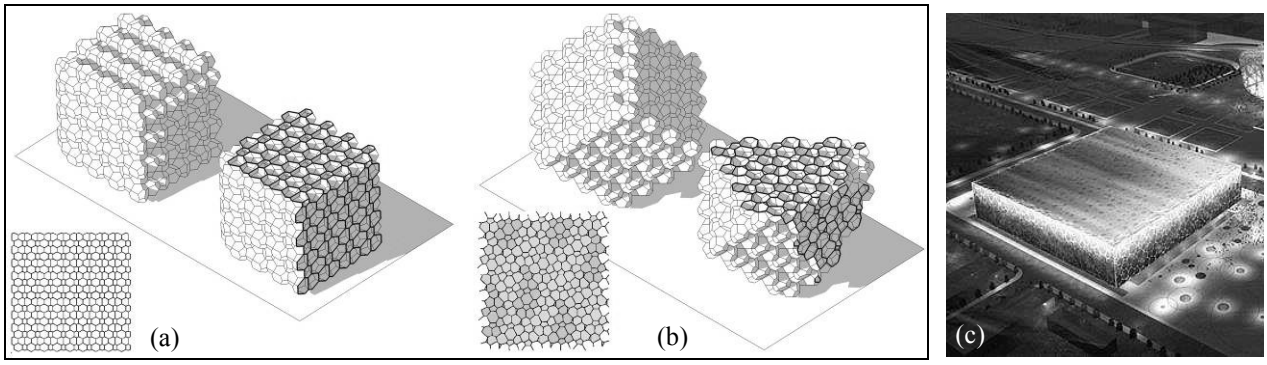
If we divide a space into many units of same volume, how can we get every unit which has most efficient structure and least contact plane? In 1873, Kelvin[1] pointed out this question (this question is equal to the question that what is the maximum efficient form of bubble). He also presented the Kelvin unit structure has 14 surfaces. There are two different kinds of polygon in Kelvin unit. The one is square (amount is 6), and the other one is hexagon (amount is 8). It is shown in Fig.3a as follow.

In fact, the spatial structure of bubble is same with some rapport structure such as clathrate, which is one kind of crystal chemical compound. In 1993, when two scientists, Weaire and Phelan, studied the bubble structure, being enlightened by the crystal chemical compound Na_8Si_{46}. They found that there is a more efficient structure than Kelvin unit structure. It is named Weaire-Phelan (WP) structure unit which has two different shapes with same volume. The one is dodecahedron and the other one is tetrakaidecahedron which includes 2 hexagons and 12 pentagons, as shown in Fig.3b [3]. As a matter of fact, we can find crystal chemical substances such as Mg_{32}（Za,Al）$_{40}$ [4] matching Kelvin unit structure. Hence, we can claim that the bubble form that matches Kelvin theory will be the most suitable for the structure of Water Cube. First, it has stability (similar with defined melting point of crystal). Second, its high repetition of unit can reduce the difficulty of construction. Based on the WP structure unit theory, the spatial structure of Water Cube optimizes the shape and size of basic unit, and it enlarges the micro molecule structure into the size of architecture structure.

[1]His used name is William Thomson(1824-1907). He was conferred Lord Kelvin by British Royal Household later. He is well-known for his contribution in Electromagnetism and Thermodynamics

In order to gain the random beauty of bubbles, the elevation of Water Cube makes a 60 degree rotation along C3 axis when cutting the accumulation of WP unit. [1][4] When the plane cutting the accumulation of WP unit making 60 degree rotation, the kinds of polygon is increased obviously with random order in visual, even if it still is repetition(see Fig. 4) [4]. But, it's difficult to find the elevation of Water Cube in everyday life, because bubbles won't keep their original direction to contact other things but always keep being vertical when they have contacts with other things (bubbles always keep 120 degree to contact each other)[5]. Therefore, Water Cube is not a box that is full with bubbles, but a cube that is cutted from an infinite rapport space structure of bubble.

In fact, the accumulation of WP structure unit applied in Water Cube has the same property with the orthogonal rectangular framework of reinforced concrete structure and pyramid usually used by trusses. They're all the polyhedrons which can fill the space gaplessly and repeatedly. Beside the higher efficiency, WP structure unit has more complicate shape and less symmetry. The new polyhedron space frame structure of Water Cube is a successful example for the application of infinite rapport structure in 3D space.



(a) The elevation before rotation. (b) The elevation after rotation. It is obvious that two of Water Cube elevation are all periodic structure, but The periodic of the elevation after rotation is not more obvious than the elevation before rotation. (c) Bird's eye view

Fig.4 Analysis of the Elevation of Water Cube

Conclusions

Infinite rapport structure is used in the building decoration, the building skin and the building space structure. Moreover application of the infinite rapport structure still has much potential in architecture, particularly in the application of three dimension space. If we regard Water Cube as a milestone of application of infinite rapport structure in architecture, the exploration of infinite rapport structure in 3D space to architects has just started.

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